Introduction: Towards a Long History of Computing

The aim of this paper is to propose a line of genealogical research into what I call the long history of computing, while illustrating its historiographical contours and significance. Much of the ongoing work in the history of computing is focused on the twentieth century; I propose to extend this history back to the seventeenth century, highlighting important yet overlooked continuities.

I am principally interested in a specific mode of computational practice that computer scientist Robert Moir has called "feasible computing."[1] Feasible computing is concerned with generating acceptably approximate solutions to computational problems. In that sense, it is distinct from a mode of computational practice focused on exactness, on finding the exact value of a function. This interest in exactness, and its corresponding category of completeness, has played a large role in the twentieth-century articulation of computer science as a discipline. Focused on the work of Alan Turing and his formal proof for a possible "universal Turing machine," this tradition tends to downplay the role of everyday practitioners and their approximations. The historian of computing Matthew Jones, for example, lets this interest in a tradition of *mathesis universalis* and a pre-history of formal systems structure his influential study of early modern computation, despite his efforts to bring instrument-makers themselves into the picture. Rather than the widely used feasible techniques which my project takes up, Jones instead focuses on the little (if ever) used mechanical inventions of Pascal, Leibniz and Babbage.[2] Jones's focus is explained by his particular interest in the relationship between thinking and machines. Yet one misses a huge swathe of territory in the history of computing by focusing on these machines. In other words, one must move the history of early modern computing beyond the history of early modern computing machines and towards early modern feasible techniques.

Typically, these feasible techniques are iterative methods for finding the approximate value of important points in differentiable functions. Seen differently, they are represented by functions whose

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[1] Robert Moir, "Feasible Computation" in *Physical Perspectives on Computation, Computational Perspectives on Physics*, ed. Michael E. Cuffaro and Samuel C. Fletcher (Cambridge: Cambridge University Press, 2018).

[2] Matthew Jones, *Reckoning with Matter: Calculating Machines, Innovation and Thinking about Thinking from Pascal to Babbage* (Chicago: The University of Chicago Press, 2016). It is worth mentioning Jones's excellent work to include the engineers who built these machines in his story, but my issue is with the topics he chooses to work on more than the methods by which he investigates them.

graphical solutions are characterized by curves rather than straight lines. Starting with a best guess, the practitioner repeatedly computes successive answers to achieve more accurate results, until a close-enough approximation has been achieved. Moir argues that these techniques of feasible computation are irreducibly inductive—that, beyond the guess that begins the process, what counts as a "good enough" approximation is not deducible from the premises of the problem but is instead determined imaginatively by the practitioner.

Moir traces "the historical origins of the … method to techniques developed by physicists to overcome the computational limitations of the mathematical formulation of theories and models of natural phenomena."[3] Regarding mathematical theories, he is interested in a set of methods in numerical analysis including Taylor series expansions and Newton's iterative methods for finding roots and critical values.[4] The other set of techniques discussed by Moir, more closely related to modeling physical phenomena but similarly approximative, are perturbation methods, like those deployed during the 1750s by Nicole-Reine Lepaute, Alexis Clairaut and Jérôme Lalande in their efforts to predict the return of Halley's comet (they were a month off). While Newton had intuited the usefulness of such a technique in his work on the three-body problem it was Euler who developed its early analytical method.[5] As Moir persuasively demonstrates, this suite of approximative methods, deployed in both theory and observation, was a central part of the mathematical toolkit of seventeenth- and eighteenth-century physicists. From algebraic calculus to astronomical observation and prediction, this tradition of mathematical physics was built on computational methods concerned more with feasibility than with exactness or completeness.[6]

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[3] Moir, "Feasible Computation," 174.

[4] For Taylor series, see Dirk Jan Struik, *A Source Book in Mathematics, 1200-1800* (Princeton: Princeton University Press, 1969), 332. For Newton's relation to series expansion, see Morris Kline, *Mathematical Thought from Ancient to Modern Times* (Oxford: Oxford University Press, 1972), 354.

[5] David Alan Grier, *When Computers Were Human* (Princeton: Princeton University Press, 2013), especially chapter 1, "The First Anticipated Return: Halley's Comet 1758." For Newton's work with perturbation, see C. M. Linton, *From Eudoxus to Einstein: A History of Mathematical Astronomy* (Cambridge: Cambridge University Press, 2004), 277. For an explanation of Euler's theory, see Moir, "Feasible Computing," 181.

[6] This tradition has ancient roots. The Babylonians were aware of a special case of Newton-Raphson, concerned with calculating square roots. Hero of Alexandria published a description of this method in his *Metrica*, and it is suspected that Archimedes had developed it as well.

Moir emphasizes that "the feasible computing strategy is likely to be replicated widely in scientific practice … specifically because the motivation for the method is epistemological: since scientists are constantly confronted with the computational limitations of their own conceptual tools, they have to find reformulations of problems that make reliable inference feasible." Thus feasible computing is "a fundamental part of a great deal of scientific inference, as well as at the core of advanced algorithms for solving problems in computational science."[7] I agree with Moir that it seems clear that these feasible techniques, in a generic sense, are a crucial piece of the foundation of modern science, and that their ubiquity is evidence of that importance. In extending the historical dimensions of Moir's argument (which principally ends in the nineteenth century) through the early developments in the history of digital electromechanical computing between 1918 and 1946, I found that his hypothesis was spot-on: early practitioners in this movement, from LJ Comrie and Wallace Eckert to Herman Goldstine and John von Neumann, were principally concerned with mechanizing these specific numerical methods.[8] Von Neumann, who is largely credited with developing the dominant technical model of the modern computer, known as the Stored-Program Paradigm (or the von Neumann architecture), placed concerns with feasibility above all else:

> Our problems are usually given as continuous-variable analytical problems, frequently wholly or partly of an implicit character. For the purposes of digital computing they have to be replaced, or rather approximated, by purely arithmetical, 'finitistic', explicit (usually step-by-step or iterative) procedures. The methods by which this is effected, i.e. our computing methods in the generic sense, are conditioned by what is feasible, and in particular more or less 'cheaply' feasible, with the devices available now. The concept of effectiveness, in fact the very concept of 'elegance', of our computing techniques is fundamentally determined by such practical considerations.[9]

These techniques of approximating solutions to taxing and burdensome problems remain a central concern in twenty-first-century computing. For example, a significant portion of the work being done in machine learning is concerned with these iterative methods aimed at progressive increases in accuracy,

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[7] Moir, "Feasible Computing," 174.

[8] See my seminar paper written for Sam Fletcher, "Feasible Computation and Approximate Problem Solving: Mechanizing Scientific Computation, 1918-1946," available on request.

[9] John von Neumann and Herman Goldstine, "On the Principle of Large Scale Computing Machines" in A. H. Taub (ed.), *John von Neumann: Collected Works Volume 5: Design of Computers, Theory of Automata and Numerical Analysis* (Oxford: Pergamon, 1961).

and approximate techniques in numerical analysis remain the fundamental tools for solving partial differential equations.

Moir is right to call attention to their historical importance to the field, but his focus on the relationship between feasible computing and mathematical physics is historically limited. As the bulk of this paper demonstrates, the astronomical and mathematical concerns of natural philosophers were certainly important areas for the development of feasible computing methods. However, the central argument of my larger project is that the use and history of these techniques significantly outstrips the history of science and mathematics narrowly conceived. This is not to suggest that these techniques are less important to science and mathematics than Moir claims. Rather, I hope that an investigation into the wider history of feasible computing can help solve a key puzzle in the history of science regarding the relationship between mathematical practice and its sociocultural contexts and manifestations in the seventeenth century.

Many historians have placed the mathematical developments of the seventeenth century at the center of their arguments about the Scientific Revolution.[10] However, despite significant advancement in our historiographical understanding of the early modern sciences over the last 75 years, the relationship between mathematical practices and social forces remains opaque. H. Floris Cohen, in the final pages of his masterful historiographical compendium on the Scientific Revolution, concluded that:

> the problem of how to connect the process of the mathematization of nature with whatever causal agent seems to present itself (in particular, if taken from the surrounding society) has proved almost wholly intractable so far, whereas a variety of such causal agents could be connected to the 'Baconian' sciences in an enlightening manner.[11]

I believe that an expansive history of the development of techniques in feasible computing over the course of the seventeenth century can clarify this puzzle by extracting the history of computation from the history of mathematical physics and rejoining it to the history of political economy, including the cultural, political and religious forces at play. This paper marks the beginning of such an effort, in the form a

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[10] E. J. Dijksterhuis, *The Mechanization of the World Picture* (Oxford: Clarendon Press, 1961), and Alexander Koyré, "Galileo and the Scientific Revolution of the Seventeenth Century" in *Metaphysics and Measurement: Essays in the Scientific Revolution* (Cambridge: Harvard University Press, 1968), 1-15.

[11] H. Floris Cohen, *The Scientific Revolution: A Historiographical Inquiry* (Chicago: The University of Chicago Press, 1994), 504.

history of the most fundamental technique in feasible computing in use between the early seventeenth century and the beginning of the twentieth: the logarithm.

Between Prediction and Prophecy: Logarithmic Divinations

One of Moir's favorite examples of feasible computing is the act of calculating by logarithmic table. He claims that "logarithmic tables are such a nice example [of feasible computing] because the pattern of transforming the problem iteratively, computing the solution, and back-interpreting the result is so clear." "We see from this process," he continues "all of the features of the general pattern of feasible computation ... thus, the use of logarithmic tables provides an early example of the pattern of feasible computation." Moir begins his own history with the claim that "this method of calculation was developed in the seventeenth century by John Napier, Joost Bürgi and Henry Briggs, and used logarithms to convert multiplication, division, exponentiation, and root extraction problems, respectively, into addition, subtraction, multiplication, and division."[12] Regarding this history of the logarithm, Moir cites none other than Herman Goldstine, John von Neumann's close collaborator on mathematical methods for mechanical computing. That Goldstine, a pioneer of the mathematics of digital electronic computing, began his own *History of Numerical Analysis* with a discussion of the logarithm makes clear the central importance of this technique to the imagination of early twentieth-century computer scientists.[13]

Since their inception, Napier's logarithmic achievements have been widely celebrated. Kepler, writing in 1624 to his friend named Krüger about recent advancements in mathematics, stated that he "consider[s] nothing to be superior to Napier's method."[14] David Hume would describe Napier as "the famous inventor of the logarithms, the person to whom the title of a GREAT MAN is more justly due than

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[12] Moir, 185

[13] Herman Goldstine, *A History of Numerical Analysis from the 16th through the 19th Century* (New York: Spring-Verlag, 1977).

[14] Michael Gottlieb Hanschius [Hansch] (ed), *Epistolæ ad Joannem Kepplerum mathmaticum Cæsareum scriptae insertis ad easdem responsionibus Kepplerianis, quotquot hactenus reperiri potuerunt* (Frankfurt, 1718), 460, cited in Brian Rice, Enrique Gonzalez-Velasco and Alexander Corrigan, *The Life and Works of John* Napier (Cham: Springer International Publishing, 2017), 32.

to any other whom his country ever produced."[15] David Eugene Smith, president of the Mathematical Association of America (1920) and the History of Science Society (1927), argued that "it is unquestionably true that the invention of logarithms had more to do with the use of decimal fractions than any other single influence."[16] Napier's legacy perhaps reached its apogee at this moment in the early twentieth century. The tercentenary of the publication of the *descriptio* in 1914 was marked with an extravagant exhibition hosted at the University of Edinburgh by E.T. Whittaker, whose *Calculus of Observations* (1924) was the paradigmatic text in numerical methods before the outbreak of WWII.[17] The exhibition's published proceedings, *Modern Instruments and Methods of Calculation: A Handbook of the Napier Tercentenary Exhibition* (1914), has been described by historian of computing Aristotle Tympas as "nothing short of a Bible, I think, for us historians of computing technology."[18]

Despite the consensus around the epochal significance of Napier's work and legacy, key facts about his life remain unknown. In dealing with these unknowns, most scholars act like Moir and err on the side of caution, discussing a generic early seventeenth-century development of logarithmic practice spread out across a few key practitioners—Napier, his collaborator Henry Briggs, and the occasional inclusion of Joost Bürgi, an associate of Brahe and Kepler who worked on similar problems. Historian of mathematics D. T. Whiteside, playing devil's advocate in a speech delivered to a 1995 conference on "Scotland's Mathematical Heritage," emphasized that it was Briggs' who convinced Napier to publish work on his logarithms. The published version was, after all, marked by Briggs' own reinvention of the technique in base 10 which made using it significantly easier for casual practitioners principally familiar with finger arithmetic—the original digital turn. These tables were outdated already by 1624, when Benjamin Baërish (also known by his latinized name Ursinus), Kepler's student and collaborator on the Rudolphine Tables, published his more accurate update to Napier and Briggs' work, the *Magnus canon*

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[15] David Hume, *The History of England* (London: T. Cadell, 1792), pp. vii. 44, quoted in Philip Almond, "John Napier and the Mathematics of the 'Middle Future' Apocalypse," *Scottish Journal of Theology* 63, no. 1, 2010.

[16] David Eugene Smith, *History of Mathematics*, vol. I (New York: Dover Publications, 1958), 244.

[17] Herman Goldstine, *The Computer from Pascal to Von Neumann* (Princeton: Princeton University Press, 1972), 287.

[18] Tympas, Aristotle, "Re: Percy Ludgate Regarding a 'True Automatic Calculating Machine,'" Message to SIGCIS mailing list (2 January, 2021), E-mail.

triangulorum logarithmicus.[19] Whiteside ended his speech with a riddle: "I leave you with an iconoclastic query. Allowing that Napier coined the name 'logarithm' itself, are we right to talk of his 'creation' of the theory of such 'ratio-numbers' when he no more invented it than John Logie Baird the cathode-ray tube which we stare at each time we watch T.V.?"[20] While the tables changed and the technique developed, it was indeed Napier who gave this manner of computation its lasting name—the logarithm. So how exactly did Napier contribute to this history? And if it was others who made the important innovations, who were they and what were their concerns?

I argue that Napier is indeed a central figure in this history. In an attempt to develop an understanding of Napier's logarithmic inspirations, with a particular interest in the etymology of the word logarithm, I speculate on a few of the prominent mysteries related to Napier. One of the defining features of the word "logarithm" is not, as Whiteside suggests, its literal translation as "ratio-number," but its relation to practices in divination via its connection to the work of the Wittenberg-based scholar Caspar Peucer, a correspondent of Brahe and a student of Erasmus Reinhold. That early modern astronomers were so quickly compelled by the spirit of Napier's ideas helps to reveal the curious epistemic status of approximative mathematics at the turn of the seventeenth century—as a practice located somewhere between prophecy and prediction. Indeed, I hypothesize that it was the development of these techniques that contributed to the separation of these categories themselves. In order to understand the intellectual environment that inspired Napier's logarithmics, one must attempt to solve some of these mysteries. The aim of this excursion into fine detail is to outline the links between Napier's mathematics and Kepler's astronomy as clearly as possible, in an effort to demonstrate the centrality of feasible computation to the emergence of the new astronomy. Such a story makes clear the importance of feasible computing methods to the history of 17th century science, and at an earlier point than scholars, including Moir, have previously suggested.

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[19] D. T. Whiteside, "And John Napier Created Logarithms …" Journal of the British Society for the History of Mathematics, vol. 29, no. 3 (2014), 154-166.

[20] Whiteside, "And John Napier Created Logarithms …", 163. Although Whiteside ends with this question, he gives his own answer earlier in the paper: "I hope no one will any more deny Napier his place as inventor of the natural logarithm, even if he never knew what its base was," 161.

Mystery I: Where was Napier educated?

The first mystery that needs to be solved to clarify our understanding of Napier's life and work is the question of his education. In 1563, at the age of thirteen, Napier disappears from the historical record and does not reappear until eight years later, in 1571. Where did he go? Although the question remains without a conclusive answer, I will advocate for one possible scenario to be considered as the most likely.

We know that Napier's mother, Janet Bothwell, who died in 1563, left a debt to "Johhne Rutherfurde," the Principal of St Salvator's College at St Andrews University, to take charge of Napier and have him board at the school. We also know that Napier did not stay at St Andrews for long, as no record of his name can be found there after his matriculation in 1563.[21] We know that Napier's maternal uncle, the Bishop of Orkney Adam Bothwell, had urged Napier's father to send him to the continent, writing in a letter: "I pray you, Schir, to send your sone Jhone to the schuyllis; oyer to France or Flandaris; for he can leyr na guid at hame, nor get na proffeit in this maist perullus wordle, that he may be savet in it, that he may do frendis efter honnour and proffeit as I dout not bot he will."[22] That Napier spent these eight years on the continent is thus the general consensus, but where on the continent did he go?

The earliest source that addresses this question comes a century after Napier's death, in George Mackenzie's *The lives and characters of the most eminent writers of the Scots nation* (1722). Mackenzie claims:

> Our Author had no sooner finished his Studies in Philosophy at St Andrews, but he was sent to his Travels by his Parents; and having stayed for some Years in the low Countries, France and Italy, he returned to his Native Country, and applyed himself closely to the Study of the Mathematics, in which he excelled all the Mathematicians of his Age.[23]

This lends credence to the idea that Napier did make a journey to the continent, but it clearly is missing crucial details. Some scholars have speculated that he wound up at a continental university, and the excellence of Napier's knowledge (particularly his distinguished Greek, which was not nearly as widely taught in Europe at the time as Latin) upon his return to Scotland in 1571 suggests that he must have been

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[21] Rice, Gonzalez-Velasco and Corrigan, *The Life and Works of John Napier*, 10.

[22] Mark Napier, *Memoirs of John Napier* (William Blackwood, 1834), 67.

[23] George Mackenzie, *The lives and characters of the most eminent writers of the Scots nation* (Printed for William Adams, 1722), Vol. 3, 519.

at a top school. Paris has been casually suggested (a few of prominent Scotsmen of the sixteenth century were Paris graduates), as have the low countries and even Italy (which seems unlikely, given Napier's devout commitment to Protestantism).[24] However, no historian who has looked at the universities in these places has found any record of Napier's attendance.

Given the lack of evidence that he was at any of these top universities, I want to defend Napier biographer Julian Havil's speculation that Geneva is the most likely place that Napier spent these eight years. The key figure in this story is St Salavator college alumnus Henry Scrimgeour, a continental merchant of rare books.[25] Scrimgeour had moved to Geneva in 1561 to begin his new job as a prominent participant in (recently converted Lutheran) Ulrich Fugger III's efforts to assemble a public library in that city. Historian Marie-Claude Tucker claims that Scrimgeour, more than any other individual, is responsible for the Greek, Latin and Hebrew manuscripts assembled in what is now known as the Fugger Collection. As Havil notes (although, frustratingly, without citation), "it is recorded that [Scrimgeour] received a young Scotsman into his home" during his residency in Geneva, which lasted until Scrimgeour's death in 1572. Without smoking-gun type evidence it is impossible to know for sure, but here are a few reasons why Scrimgeour is a likely candidate to be Napier's mentor during this eight-year absence from Scotland.

One of the most compelling reasons is Scrimgeour's connections to Continental Greek studies—the central pillar of Napier's work on the apocalypse (discussed below) is a close reading of St. John's Greek. Scrimgeour, after beginning his Continental studies in 1538 at the University of Paris under Petrus Ramus, moved to Bourges where he studied Law. It was as a scholar of civil law that Scrimgeour made his reputation—his 1558 edition of *Justinan's Novels* was his first work under the patronage of Ulrich Fugger III, and it was well received.[26] While a student at Bourges, Scrimgeour made friends with Jacques Amyot, a professor of Greek whose translations of Plutarch into French would be the basis of Sir Thomas

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24 Rice, Gonzalez-Velasco and Corrigan, *The Life and Works of John Napier*, 12.

25 Julian Havil, *John Napier: Life, Lograithms and Legacy* (Princeton: Princeton University Press, 2014).

26 Marie-Claude Tucker, "Scrimgeour [Scrymgeour], Henry," *Oxford Dictionary of National Biography* (Oxford: Oxford University Press, 2004).

North's English translation, the source of Shakespeare's Roman plays.[27] By 1548, Scrimgeour had succeeded Amyot as the mentor of the three sons of Guillaume Bochetel, the French Secretary of State, evidence that the man was open to playing the role of a tutor.

Scrimgeour was appointed a reader in philosophy at the University in Geneva in 1563, although it was remarked that he was often absent on journeys to purchase books. That Fransicus Portus, a renowned scholar from Crete, joined the University of Geneva in 1562 as a professor of Greek is also worth mentioning, and it is likely that Scrimgeour communicated with Portus about the Greek manuscripts he was buying for Fugger's library. Napier, who returned to Scotland in 1571 "sufficiently versed in Greek to offer corrections to the Vulgate of St Jerome," had clearly received an excellent education in Greek, and without evidence of his formal enrollment in any of the premier universities that offered Greek, perhaps an informal education alongside Scrimgeour is the most likely alternative.[28] More research focused on Scrimgeour himself (there is none beyond short biographical entries) could do much to confirm or deny these suspicions.

Another interesting piece of evidence is that in 1570, the year prior to Napier's reappearance in the historical record—he was married in Scotland in December of 1571, and he remained in that country afterwards—the Earls of Moray and Mar sent a letter to Scrimgeour asking him to return to Scotland to become the tutor to the young King James VI, an offer Scrimgeour turned down because of the dangerous nature of Scottish court politics (indeed, the Earl Moray would be murdered that very year) and his advanced age.[29] Perhaps news of Napier's excellent education inspired such a request; or Napier's journey back to Scotland might have provided the elderly Scrimgeour with a trustworthy travel companion.

Napier's uncompromising Protestantism is a kind of mystery in itself. Napier's father Archibald was a Protestant, but the extent of Napier's conviction cannot reasonably be attributed to the man. An education in Geneva during the immediate aftermath of Calvin's death, however, might have primed the young Napier for his overwhelming commitment to protestant eschatology and geopolitics. Scrimgeour

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[27] Hugh Chisholm, ed., *Encyclopedia Britannica*, vol. 1 (Cambridge: Cambridge University Press, 1911), 901.

[28] Havil, *John Napier*, 16.

[29] Marie-Claude Tucker, "Scrimgeour [Scrymgeour], Henry," *Oxford Dictionary of National Biography* (Oxford: Oxford University Press, 2004).

himself was witness to Calvin's last will when the theologian died in the Spring of 1564, so Napier would have likely been involved in this circle. Finally, a few other young Scotsmen wind up in this area between 1560 and 1575: Andrew Melville (Scrimgeour's nephew and another well-known Scottish scholar) became a professor at Geneva in 1569, and Napier's good friend John Craig, the Scottish physician and polymath and a central figure in this story of logarithmics, attended the nearby University of Basel for some time during the 1570s.[30]

Mystery II: When did Napier develop his logarithms?

It is John Craig who provides, albeit obliquely, the key insight to the second Napier mystery: when did Napier begin developing the logarithm? The first published account of Napier's logarithmics is the 1614 *Descriptio*, which had already been meaningfully transformed by Henry Briggs' introduction of base 10. How long had Napier been sitting on his method? Answering this question is the key to unlocking the context in which Napier's logarithmic work began, as well as helping to differentiate Napier's original technique from Briggs' streamlined version published in *Descriptio*. A 1624 letter from Kepler to a friend (the beginning of which I cited above) gives the evidence for the earliest mention of Napier and his logarithms: "Moreover, I consider nothing to be superior to Napier's method: even though a certain Scotsman [John Craig], in a letter written to Tycho in the year 1594, already expressed hope of that wonderful Canon."[31] Mark Napier, the compiler of the *Memoirs of John Napier*, suggests that Craig accompanied King James VI on his trip to Denmark to marry Queen Anne in 1590, a trip that included a one day stop at Brahe's observatory—Mark Napier goes as far as to claim that the detour was entirely Craig's idea.[32] While no direct evidence of Craig's trip to Denmark exists, he was close to the King. Craig

[30] John Henry, "Craig, John (d. 1620?)," *Oxford Dictionary of National Biography* (Oxford: Oxford University Press, 2004).

[31] Michael Gottlieb Hanschius [Hansch] (ed), *Epistolæ ad Joannem Kepplerum mathmaticum Cæsareum scriptae insertis ad easdem responsionibus Kepplerianis, quotquot hactenus reperiri potuerunt* (Frankfurt, 1718), 460, cited in Brian Rice, Enrique Gonzalez-Velasco and Alexander Corrigan, *The Life and Works of John* Napier (Cham: Springer International Publishing, 2017), 32.

[32] John Henry, "Craig, John (d. 1620?)," *Oxford Dictionary of National Biography* (Oxford: Oxford University Press, 2004).

became James VI's first physician in 1603 (a title Craig shared with the German Martin Schöner), and travelled with him in that year to England where the monarch was to take Elizabeth's throne.

It has been suggested that Craig's contact with Brahe was facilitated by the Scottish diplomat William Stewart of Houston, who was the principal courier of letters between James VI and Queen Anne during their courtship.[33] While Houston could have helped Craig facilitate a visit to Uraniborg as the general organizer of the King's 1590 journey to Denmark, it is likely that Craig already had an in with Brahe via the Scotsman Duncan Liddel, to whom Craig had taught mathematics while serving as a professor at the University of Frankfurt around 1580. Liddel, who remained in North Germany until 1607, visited Tycho Brahe during both 1587 and 1588. The two were originally friends, and Liddel's companion Johannes Caselius (another professor at the University of Helmstadt, where Liddel was based) claimed that Liddel was "the first in Germany to teach astronomy according not only to the theories of Ptolemy and Copernicus, but also to Brahe's outline." Brahe, apparently, did not take kindly to this and accused Liddel of plagiarism, though Liddel insisted he had always given Brahe credit for his idea.[34] But the two were likely still on good terms before the James VI's visit to Denmark in 1590.

Dating Craig's contact with Brahe helps to solve this second Napier mystery, regarding the time frame of his original work on the logarithm. If one assumes that Craig had been in contact with Brahe in 1590, and that it is not until 1594 that Craig mentions to Brahe Napier's work on logarithms, then it is likely that it was during this span of years in the early 1590s that Napier developed his technique. Additionally, if it is this Craig-Liddel-Brahe connection that carried Napier's method to Brahe, then this is a crucial clue to the third Napier mystery: Napier's inspiration for developing the mathematics of the logarithm.

Mystery III: What was the mathematical inspiration for Napier's work?

The common academic denominator between John Craig and Duncan Liddel is the German astronomer and mathematician Paul Wittich. Craig became Wittich's student at Frankfurt in 1576, and Liddel moved

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[33] Andrew Pyle, ed., *Dictionary of Seventeenth Century British Philosophers* (200), 218-19.

[34] Charles Platts, revised by George Molland, "Liddel, Duncan," *Oxford Dictionary of National Biography* (Oxford: Oxford University Press, 2004).

to study under Wittich around 1583, on the advice of Craig, after Craig returned to Scotland. Wittich had lived with Brahe for a time at Uraniborg during 1580, and so represents another strong link in this network. Wittich is important because of his familiarity with an obscure mathematical technique of the sixteenth century called prosthaphaeresis. In fact, Wittich is said to have taught Brahe this method.[35] The reason why prosthaphaeresis is relevant to the development of Napier's logarithms is that it offered a more tedious and complex method of doing essentially what Napier's logarithms would do—approximating the solution of computationally expensive problems in multiplication and division by more feasible means, in this case a trigonometric shortcut. The story of prosthaphaeresis, which is worth telling in some depth, lies beyond the scope of this essay. Although no concrete evidence of the influence of this practice on Napier's development of the logarithm has been established yet, if my above exercise in historical riddle-solving is close to a truth, such evidence may indeed be out there waiting to be found. In any case, it is likely that Napier, via his friendship with Craig, was aware of prosthaphaeresis.[36] That Craig's own copy of Copernicus's *De revolutionibus* includes annotations on prosthaphaeresis strengthens this possibility, indicating that Craig was familiar with the technique.[37] It also serves to link Napier's project to near contemporaneous work being done by Brahe's associate Joost Bürgi, and there is solid evidence that Bürgi's work was inspired directly by prosthaphaeresis, as he was an innovator of that technique after being introduced to it by Wittich on the latter's visit to Kassel in 1584.[38]

The earliest discussion of Napier's possible encounter with the mathematics of Brahe and Kepler comes from Anthony Wood's biographical dictionary of prominent men of Oxford, the *Athenae Oxonienses* (1691–2), in the entry for Henry Briggs:

> It must be now known, that one Dr. Craig a Scotch man … coming out of Denmark into his own country, called upon Joh. Neper baron of Marcheston near Edinburgh, and told

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[35] V. Thoren, "Prosthaphaeresis Revisited," *Historia Mathematica*, vol. 15 (1998), 32-29.

[36] For more on prosthaphaeresis, see Harvard's Oliver Knill's hosting of Brain Bothers' short publication on the subject: http://people.math.harvard.edu/~knill/history/burgi/prost.pdf. See also Klaus Kuehn and Jerry McCarthy, "Prosthaphaeresis and Johannes Werner (1468-1522)," also published on the web: http://www.oughtred.org/jos/articles/PROSTHAPHAERESISandWERNERfinal.jmccLR8.8.pdf

[37] Owen Gingerich, *The Book Nobody Read: Chasing the Revolutions of Nicolaus Copernicus* (New York: Walker Publishing Company, 2004), 103-106, cited in Rice, Gonzalez-Velasco and Corrigan, 396.

[38] Denis Roegel, "Bürgi's 'Progress Tabulen' (1620): Logarithmic tables without Logarithms" (LOCOMAT project, 2010).

> him among other discourses of a new invention in Denmark (by Longomontanus as 'tis said) to save the tedious multiplication and division in astronomical calculations. Neper being solicitous to know farther of him concerning this matter, he could give no other account of it, than that it was by proportionable numbers. Which hint Neper taking, he desired him at his return to call upon him again. Craig, after some weeks had passed, did so, and Neper then shew'd him a rude draught of what he called, *Canon mirabilis Logarithmorum*. Which draught, with some alterations, he printing in 1614, it come forthwith into the hands of our author Briggs, and into those of Will. Oughtred, from whom the relation of this matter came.[39]

While some of the material in this account has been shown to be inaccurate, and the inclusion of Longomontanus is almost certainly apocryphal (Wittich is by far the more likely candidate), Wood's account is the best we have about the possible origins of the logarithm in prosthaphaeresis.

Thus far, three Napier mysteries have been conjectured upon: his education (Geneva), the time frame of his original work on the logarithm (1590–1594) and the source of his mathematical inspiration for the technique (prosthaphaeresis). One mystery remains, and it is the one that Whiteside had singled out as the most important as it relates to one of Napier's clearest contributions to logarithmics: its name. Why did Napier name his technique the logarithm? And what did such a name mean to those early influential adopters, such as Brahe and Kepler? This is where I want to make my most important argument: that logarithmics occupied a complicated epistemic position located somewhere between prophecy and prediction.

Mystery IV: Why did Napier name his technique the logarithm?

Napier spent the period 1588–1593 furiously at work on the only text he ever felt truly compelled to write, and which earned him a celebrity status in his lifetime. *The Plaine Discovery of the Whole Revelation of St. John* was the culmination of Napier's humanistic learning and his Protestant fervor. In the text, Napier's close reading of St. John's Book of Revelations was combined, via an iterative computation, with a history of the world, in an effort to predict when Christ might return to bring the Kingdom of God to earth. This text was a major success, and had additional English printings in 1594, 1611 and, crucially, 1645. It also had editions printed in Dutch (1600 and 1607), French (1602, 1603, 1605 and 1607) and German (1611, 1612, 1615 and 1627). As this was Napier's only published work

[39] Anthony A. Wood, *Athenae Oxonienses*, vol. II (London, 1815), 492.

before the 1614 *Descriptio*, it is clear that his reputation, at home and abroad, would have been tied most closely to *this* eschatological study. Napier was not an astronomer, but nonetheless he was a well-respected practitioner of mathematics, known for his predictions.

I argue that one can read Napier's *Plaine Discovery* as an exercise in approximate problem-solving that mirrors the structure of feasible computing as Moir describes it. Like computing with a logarithmic table, Napier's reading of revelation attempts to "transform the problem iteratively, compute the solution, and back interpret the result." The first step, the transformation of the problem, occurs in propositions 15 and 16, where Napier takes a complicated Biblical chronology, composed of a heterogenous collection of temporalities expressed in symbolically dense periods of days/months/years in both time and "half times," and compacts it into a uniform one: each prophetical day is equivalent to one year in real time. The historian and philosopher of religion Phillip Almond explains: "This is to the effect that the 42 months, 1,260 prophetical days, three great days and a half, and times, times, and half a time mentioned in the books of Daniel and Revelation are all one time. This is supplemented by proposition 16 which is to the effect that all of these times signify 1,260 years."[40] After a few more transformations and computations, Napier concludes that there should be about 1,512 years between the "First Seal" and the "First Jubilee." With the transformation completed, and solution computed, all that remains is for Napier to "back interpret the result." Here, Napier attempts to match the math of his chronology (which, again, he approximates to be 1,512 years long) with historical events that could reasonably be the ones symbolically prophesied by John and Daniel. Thus, the "First Seal" becomes the baptism of Christ, which Napier puts at 29 AD. If this is the case, then the "First Jubilee" must be around 1541, which Napier identifies generically in the text with the Protestant movement taking root in Europe, though it is worth noting that 1541 is the year that Calvin was invited to Geneva to establish his church.[41] Napier's cryptic

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[40] Philip Almond, "John Napier and the Mathematics of the 'Middle Future' Apocalypse," *Scottish Journal of Theology* 63, no. 1, 2010.

[41] Rice, Gonzalez-Velasco and Corrigan, 77 fn. 32

identification of 1541 as *the* important date in European Protestantism takes on new importance if Geneva was indeed the site of his education, and Scrimgeour his tutor, and thus confirming or denying this Calvinist connection would be a crucial first step to further elaborating Napier's theological beliefs.

If it was during Napier's writing of the *Plaine Discovery* that he also had his logarithmic breakthrough, as the date range of 1590-1594 via Craig's communication with Brahe suggests, then it is reasonable to intuit some connection between his choice of name for that technique and his studies in Biblical chronology, which were his focus. The Kepler scholar Wolfgang Osterhage has suggested (although without elaboration or citation) that Napier got the name "logarithm" from Caspar Peucer, who had coined the phrase *logarithmanteia*, which Osterhage translates as "meaning something like 'prediction of word by number and vice-versa'."[42] Note the already drastically different tone between logarithm as "prediction of word by number and vice-versa" from Whiteside's literal reading of the phrase as "ratio-number." The act of induction central to the feasible technique, made clear by Osterhage's use of the word prediction in his translation of *logarithmanteia*, is smothered by Whiteside's use of a geometric language of ratios. Following Osterhage, I argue that the predictive character of the logarithm should be emphasized over a connection to a kind of geometric ratio. Not only does this cut closer to what Napier likely had in mind when he coined the phrase, as I elaborate below, but it also emphasizes the inductive character of logarithmics rather than appealing to a kind of geometric epistemology of deduction.

Regarding the origin of the phrase *logarithmanteia*, Osterhage mentions that it appears in Caspar Peucer's C*ommentarius des praecipius divinationem generibus*, published in 1553. Osterhage generously translates the title of this work as *Commentary about Kinds of Predictions*, but it is in actuality a much more expansive tome on divination and magic in general.[43] While Osterhage does not detail how or why Napier would have encountered this term and appropriated it, and Peucer himself is not mentioned by any of the above-mentioned Napier biographers, I think that Osterhage's claim is indeed likely, and serves to solidly connect Napier's invention of the logarithm to his prophetic work in the *Plaine Discovery*.

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[42] Wolfgang Osterhage, *Johannes Kepler: The Order of Things* (Cham: Springer, 2020), 79.

[43] Lynn Thorndike, *A History of Magic and Experimental Science* (New York: Columbia University Press, 1941), 493-502.

It is clear that Napier had read Peucer, because he cites him in his *Plaine Discovery*, on a most crucial point:

> forasmuch as the *Grecians* had a custome in their mysteries and Oracles to observe the number of names, as ye shall finde in divers parts of *Sibylla*: And as in that countrey, the name of the flood [here Napier writes the Greek name] is celebrated as holie, because it containeth the number of the daies of the yeare 365. as *Gasparaus Peucerus* upon *Carion* testifieth, *Chro. Lib. 4.* Where he describeth the *Cattes* and *Hesses*. Therefore, Saint *John* (observing the custome of them to whome her writeth) saieth that the … number of the Beasts name [again, Napier writes this in the Greek], is 666.[44]

Napier goes on to argue that the number 666 is not part of Biblical chronology, but only a description of the number produced by a numerical reading of the Beast's Greek name as given to him by John. Here Napier cites Peucer's *Chronicon carionis* (1550), published collaboratively with Protestant theologian Phillip Melanchthon.[45] It is in the *Chronicon* that Peucer makes this argument about 365 being the number of the flood, but Peucer's argument about 666 as the number of the beast comes only in *Commentarius des praecipius divinationem generibus*, so it is likely Napier was familiar with both texts. Peucer introduces what he calls a *Refutatio logarithmantae* on the same page in *Commentarius des praecipius divinationem generibus* which he describes the number of the beast—the origin, if we are to believe Osterhage, of Napier's phrase. If Napier's logarithmics were indeed developed through a close study of divination and its various practices, then the use of such approximative techniques by astronomers interested in building predictive models raises important questions about the epistemic foundations of the new astronomy. The peculiarly Protestant (and even specifically Calvinist) environment in which these ideas gestated can add a new layer to the already well-traveled ground of the history of science and religion in early modern Europe. At the very least, it seems likely that when Brahe, Kepler and their ilk adopted Napier's logarithmics, they were well aware that it came from a man known for his scriptural predictions who had no published work in either mathematics per se or astronomy. A closer investigation of this North Sea network of Scottish and German polymaths might therefore be the basis of a study into the social and cultural context of seventeenth-century mathematics.

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[44] John Napier, *A Plaine Discovery of the Whole Revelation of St John* (1593), in Rice, Gonzalez-Velasco and Corrigan, 164.

[45] Caspar Peucer and Phillip Melachthon, *Chronicon Carionis* (Geneva: Sumptibus S. Crispini, 1617). Napier is citing the discussion of the name of the flood on page 394.

There is one final sub-mystery worth considering. How did Napier come across Peucer's *Commentarius*? The text is of an older generation than Napier's, and while Peucer was a well-known member of the German astronomical community throughout the 1550s and 1560s, the man was arrested and held in prison for twelve years between 1574 and 1582, and his personal stature in the astronomical community seemed to have waned significantly after his release, though an edition of his *De dimensione terra* (1550) was published in 1587. This text itself contains a link between Peucer's measurements and Biblical prophecy, so it is not impossible that its 1587 edition came quickly to Napier and inspired a deeper interest in Peucer's work. However, the more likely scenario is that Napier had been familiar with Peucer from his time with Scrimgeour in Geneva.

First, Peucer's *Commentarius*, particularly the chapter cited by Napier in the *Plaine Discovery*, is written in a combination of Latin, Greek and Hebrew—the three languages Scrimgeour specialized in. Second, and more interestingly, Peucer became known as prominent crypto-Calvinist operating from the Lutheran stronghold of Wittenberg, where he had his university chair.[46] In fact, it was Peucer's suspected Calvinism that was the reason for his imprisonment in 1572. Given that Scrimgeour was active in Calvin's circle, proficient in Latin, Greek and Hebrew, and likely quite interested to read the work of a Wittenberg based Calvinist, it is likely that Napier was exposed to Peucer's work much earlier than 1587, and instead might have encountered these texts growing up in Geneva.

This would help to explain Napier's long-lasting interest in apocalyptic prophecy. While it is true that Napier did not start writing his *Plaine Discovery* until 1588, he had been actively engaged in the project for much longer. Napier's motivation for putting pen to paper in 1588 came from what must have seemed like an apocalyptic emergency—news of the Spanish Armada's voyage to the British Isles. As Napier writes, this news caused him to be "conftrained of compaffion, leauing the Latine, to hafte out in Englifh this prefent worke, almoft vnripe, that thereby, the simple of this Iland may be inftructed, the godly confirmed and the proud, and foolifh expectations of the wicked beaten down."[47] Thus, clearly Napier had been laboring on this project for a period before his transition to the English language in 1588,

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[46] Robert Kolb, "Review: Günter Frank and Herman J. Selderhuis, eds. *Melancthon und der Calvinismus*," *Renaissance Quarterly*, vol. 59, no. 3 (2018).

[47] Rice, Gonzalez-Velasco and Corrigan, 17.

and if after the five years he spent on it before its publication in 1593 the text was still “unripe,” perhaps Napier had already done substantial work on the topic—particularly if he had encountered Peucer in his youth, rather than through the 1587 edition of *De dimensione terra*.

My hope is that this section, through an investigation of some of the Napier mysteries, illustrates how mathematical approximations straddled the line between prophecy and prediction. The next section offers a quick look at the work of Brahe’s successor, Kepler, and his “New Astronomy” to show that these early feasible computations were not just computational aids for compiling tables, but were a crucial element at the very heart of this early episode in the Scientific Revolution.

Kepler’s Feasible Computing: Approximation and Prediction

> Traditionally, [mathematical] theories generate, rather than limit, the exactitude of the so-called exact sciences. According to this view, once the fundamental laws of a physical theory are stated in bare mathematical form, testable predictions can, at least in principle, be calculated to any level of accuracy. The only limits placed upon such calculations are those imposed by the patience and competence of the human calculator. Mathematical theory, then, is definitive of the exactitude to which precision measurement can aspire.
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> In practice, however, the process of generating testable predictions from mathematical theories is much less straightforward than is generally supposed. Cartwright has pointed out, for example, that most testable predictions are arrived at via a series of approximations whose precise form is not dictated by theory itself … Cartwright's point is, primarily, a philosophical one; she seeks to show that the chain of reasoning from fundamental to phenomenological laws is, as it were, inferentially irreversible. By the time we have deduced a testable prediction from a fundamental theory, we have made so many approximations and auxiliary assumptions that it is unclear what is being supported if the prediction turns out to be correct … The exactitude of the exact sciences derives, it seems, more from the fact that the chains of reasoning (from physical laws to precise predictions) are expressed in mathematical form, than from the epistemic security of the laws themselves.
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> Andrew Warwick, “What’s Exact about the Exact Sciences?” in M. Norton-Wise, *The Values of Precision*.

My larger academic project is principally concerned with developments in feasible computation that lie outside the domain of mathematical physics in an attempt to argue for a broader social development of early modern computing techniques. Before moving on to these non-physico-mathematical matters, it is worth considering why these techniques are fundamental to the emergence of what I argue, following

Andrew Warwick, have been inaccurately called the "exact sciences." Kepler's decisive move to imagine the orbits of the planets as elliptical rather than circular serves as a case in point.

In Chapter 16 of his *Astronomia nova* (1609), Kepler details the process by which he formed his *hypothesis vicaria*—his last model of the planets based on circular motion.[48] At first glance, Kepler's model looks traditionally Ptolemaic, with the obvious exception that he is modeling Mars's motion around the sun rather than the earth. He assumes the motion to be circular, and—using observational data collected by Brahe between 1587 and 1595—he charted a model of the planet's course. What is novel about Kepler's model, however, is that he allowed the equant to fall at an arbitrary place on the apsidal line, a dramatic innovation.

The equant is an ancient concept introduced by Ptolemy in an attempt to align his observational data with an Aristotelian physical paradigm that necessitated heavenly movement be circular, an attempt to "save the phenomena." As the centuries wore on, the equant became increasingly problematic. It became a threat to Ptolemaic astronomy as a necessary compromise between theory and experience, and a number of prominent medieval and early modern astronomers—including both Copernicus and Brahe—were deeply suspicious of it.[49] Kepler, as both a believer in Copernican heliocentrism as well as an intimate collaborator of Brahe, was well aware of its philosophical misgivings.

Ptolemy devised the equant to occupy a specific place—it was located at an equal distance from the center of the circular orbit from the earth, which was located on the opposite side of that center point. The necessity that the equant be placed there, rather than somewhere else, is what Kepler implicitly called into question when he computed the motions of Mars with an arbitrarily placed equant, decidedly closer to the center of the circular orbit than the sun, which had replaced the earth on the opposite side. How Kepler determined this location of the equant remains a mystery, although one scholar's description of his choice as "arbitrary" begs this question. Warwick mentions that these kinds of approximations are often

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48 For a modern reconstruction of Kepler's work, see Y. Maeyama, "Kepler's Hypothesis Vicaria," *Archive for the History of Exact Sciences*, vol. 41, no. 1 (1990), 53-92.

49 For a good account of Copernicus's problems with the equant, see Peter Barker, "Copernicus, the Orbs, and the Equant," *Synthese,* vol. 83, no. 2 (1990), 317-323. For a description of Brahe's criticism, see Pierre Duheim, *To Save the Phenomena: An Essay on the Idea of Physical Theory from Plato to Galileo* (Chicago: University of Chicago Press, 1969), 96-97.

not dictated by the theory, but are instead the result of "professional skill and judgement."[50] A closer look at what precise concerns motivated Kepler's decision could contribute to bringing the history of craft and artisanship, as well as what Lorraine Daston has called the "moral economy" of science, into the history of early modern computation.[51] Though not taken up here, William Ashworth's exceptional work regarding the relationship between the ethics of sound accounting and the proper methods of astronomical mathematics in the nineteenth century is a model for bringing a concern with moral economies and artisan/practitioners into the study of the intersection of computation and scientific practice.[52]

The equant, which at that point had been bound for philosophical reasons to a particular point in the model, was released by Kepler to serve instead as a helpful computational anchor in his iterative method of predicting future positions of Mars based on this model. Having determined that the problem could not be solved geometrically, Kepler deployed a feasible computing strategy based on an iterative process of nested approximations which he described as a "*duplicem falsam positionem*" (double false position).[53] It is worth noting that Kepler had earlier identified the essential medieval approximative technique, known as the *regula falsi* (the method of false position), as a model for his general approach to knowledge in his *Apologia pro Tychone contra Ursum* (1600).[54]

Kepler's *hypothesis vicaria* ultimately collapsed when it failed to correspond to a model of Mars' orbit built from his measurements of that planet's actual distance from the sun, rather than by dealing with

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[50] Andrew Warwick, "What's Exact about the Exact Sciences?" in M. Norton-Wise, *The Values of Precision* (Princeton: Princeton University Press, 1995), 312.

[51] Lorraine Daston, "The Moral Economy of Science," *Osiris*, vol. 10 (1995), 2-24. See also Pamela Long, *Artisan/Practitioners and the Rise of the New Science, 1400-1600* (Corvallis: Oregon State University Press, 2011).

[52] William Ashworth, "The Calculating Eye: Baily, Herschel, Babbage and the Business of Astronomy," *The British Journal for the History of Science*, vol. 27, no. 4 (1994), 409-441.

[53] For a description of Kepler's method as an exercise in numerical analysis and feasible computing, see Steinar Thorvaldsen, "Early Numerical Analysis in Kepler's New Astronomy," *Science in Context*, vol. 23, no. 1 (2010), 39-63.

[54] For more on Kepler's *Apologia* as it relates to his science, see N. Jardine, *The Birth of History and Philosophy of Science: Kepler's 'A Defense of Tycho against Ursus' with essays on its provenance and significance* (Cambridge: Cambridge University Press, 1984). For the reference to the *regula falsi*, see 95-96 (Latin) and 149-150 (English). Quoted at length in Thorvaldsen, 45. Note that Robert Recorde's *Grounde of Artes*, the essential sixteenth-century English arithmetical textbook, covers the regula falsi, albeit in verse rather than mathematical notation. See D. E. Smith, *The History of Mathematics*, vol. II (New York: Dover, 1958), 437-441.

the relative distances which had characterized his original approach. This shift away from relative proportionality is a remarkable development, but the word "exact" fails to capture its essence, and Kepler's move from a circular to an elliptical model of planetary motion not only preserved but expanded the role for approximative computing methods in his new astronomy. Kepler's second law, which sought to approximate the changing speed of a planet throughout its orbit via calculating an area of the ellipse, has been described by Steiner Thorvaldsen as "a kind of computational technique." Indeed, the new shape of the ellipse itself *necessitated* an approximative astronomy, as Thorvaldsen explains: "To compute areas inside the ellipse is not easy. There is no simple or exact way to find explicitly the position angle corresponding to a given time, and consequently there was no direct way to calculate the planet's position exactly from its date, as it had always been possible before Kepler."[55] Kepler's iterative solution, which appeared in Book V of his *Epitome of Copernican Astronomy* (1621), is now known as *Kepler's equation*, although he called it his "regula positionum" (rule of supposition), and Kepler was only able to solve it approximately for particular cases.[56]

Warwick's argument for the approximative nature of the exact sciences is historically focused on nineteenth-century British mathematical tables and calculating machines—and this is certainly a vital part of the story. But, as I have shown, these approximative techniques have been at the heart of the physical sciences since the time of Kepler, and likely Brahe before him. This expansion of feasible computing to the beginning of the seventeenth century is a significant addition to both the work of Moir, which begins essentially with Newton, as well as the work of Historian of Computing David Alan Grier, which begins with the invention of calculus and explicitly "overlooks several important computational projects, including the *Arithmetica Logarthmica* of Henry Briggs … and the planetary computations in the *Rudolphine Tables*."[57]

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[55] Thorvaldsen, "Early Numerical Analysis," 48.

[56] Noel M. Swerdlow, "Kepler's Iterative Solution of Kepler's Equation," *Journal for the History of Astronomy* vol. 31 (2000), 339-341.

[57] Alan Grier, *When Computers Were Human*, 6. Note that Alan Grier discounts these because they were not examples of "organized computation … none of these scientists employed a staff of computers…". Even if one uses this criteria of "organized computation" to structure the history, surely the use of logarithmic tables by excisemen would count as an important development, and it is one that Alan Grier overlooks entirely.

Kepler, in addition to his importance to the history of science broadly, is a crucial figure in my story about feasible computing and the emergence of logarithmics. An obvious connection is his *Chilias logarithmoria* (1624), in which Kepler developed his own logarithmic technique inspired by Napier's and Briggs' work. These logarithms were an essential tool for tackling the exhaustingly laborious calculations involved in his production of the *Rudolphine Tables*, calculations not inaccurately described by Arthur Koestler as "herculean donkey work."[58] Another connection is to a more obscure text, Kepler's *Nova stereometria doliorum vinariorum* (1615). This text was obscure even in its own time, as Kepler struggled to find anyone willing to publish it—an intensely mathematical treatment of wine barrel measurement complete with a description of Kepler's version of a gauging rod must have seemed niche.[59] In some respects, this text seems marginal and its first full translation into a modern language came only in 2018. On the other hand, Eberhard Knobloch (the recent translator) describes it as "[Kepler's] most important mathematical publication" and the nineteenth-century German historian of mathematics Maurtiz Cantor called it "the source and the inspiration for all later cubatures."[60] While Kepler's gauging rod does not seem to have made it to England (Kepler himself was infatuated by the perfection of the Austrian barrel, anyway), English gauging—which became a central technical operation of the state with the formation of the Board of Excise in 1643—was based entirely in logarithmics. Fleshing out this connection, mediated by logarithmics, between Kepler's astronomy and his barrel gauging, and connecting it to developments in England, can add significantly to our understanding of early modern feasible computations at the intersection of science and the state.

Conclusion: Towards a seventeenth-Century English history of computation

Kepler's experience with the logarithm would prove decisive, allowing him to greatly speed up his work on the Rudolphine Tables—one of the key early-seventeenth-century achievements in applied feasible

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[58] Arthur Koestler, *The Sleepwalkers* (London: Hutchison, 1959), 401, mentioned in Elizabeth Eisenstein, *The Printing Revolution in Early Modern Europe, Second Edition* (Cambridge: Cambridge University Press, 2005), 246.

[59] See Johannes Kepler, *New Solid Geometry of Wine Barrels, A Supplement to the Archimedean solid Geometry has been Added*, trans. Eberhard Knobloch (Paris: Les Belles Lettres, 2018), 20.

[60] Mauritz Cantor, *Vorlesungen über Geschichte der Mathematik*, vol. II (1894), 863. Translated in Thorvaldsen, "Early Numerical Analysis in Kepler's New Astronomy," 40.

computing methods—and by the end of the century figures such as Newton further developed numerical analysis as a fundamental component of their approach to mathematical physics.

These developments, however, were just one aspect of a near-total penetration of logarithmics into the social, economic and political world of early modern England over the course of the century. As I have suggested, Napier's arcane approach was made significantly more feasible by the intervention of Briggs and the publication of the first "digital" logarithms in base 10 in the 1614 *Mirifici logarithmorum canonis descriptio*, and after that the flood gates opened. Edward Wright's English translation of the *descriptio*, published in 1616, was dedicated to the East India Company. As Wright notes in his dedication, his translation of Napier's text could help a company whose "imployment of so many Mariners in so many goodly and costly ships, in long and dangerous voyages, for whose use (though many other wayes profitable) this little booke is chiefly behoouefull."[61] This was the start of a long trend, as computation by logarithmic table remained the principle means of the construction of naval almanacs until L. J. Comrie's tenure as head of the Nautical Almanac Office in the 1920s, when they were replaced by a Hollerith punched-card tabulator.[62] Additionally, the work of the English instrument maker Edmund Gunter, who made decisive contributions to the slide rule at this early moment around 1620, deserves significantly more scholarly attention.[63]

By the 1620s, logarithmic tables were put to use by the Church of England for assessing "fees" to be charged to those who rented their land. The most influential early work in this kind of computation was done by Ambrose Akryod, whose 1628 *Tables of Leasses and Interest, with Their Groundes Expressed in Four Tables of Fractions* held sway until George Mabbut's publication of *Tables for Renewing & Purchasing of the Leases of Cathedral-Churches and Colleges* in 1686—a text whose later editions would

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61 Edward Wright, *A Description of the Admirable Table of Logarithmes* (1616).

62 Mary Croarken, *Early Scientific Computing in Britain* (Clarendon Press: Oxford, 1990), 36.

63 Florian Cajori, *A History of the Logarithmic Slide Rule and Allied Instruments and on the History of Gunter's Scale and the Slide Rule during the Seventeenth Century* (Mendham: Astragel Press, 1994 [1909]).

be wrongly attributed to Newton, due to his endorsement of them. It is likely that Akryod was introduced to Briggs work with Napier on the logarithm via Akryod's position as a member of Trinity College's financial administration between 1615 and 1625.[64] That Euler's number (*e*), the base of the natural logarithm, was stumbled upon in 1685 by Jacob Bernoulli through his attempts to compute continuous compound interest via series expression shows the lasting importance of financial concerns to the development of logarithmics.[65]

Edmund Halley's famous effort to mathematically audit the Parliament's issuance of the 1693 Millions Act—which attempted to raise one million pounds via a tontine and the issuance of life annuities on one, two, or three lives—was made singularly feasible by logarithmics.[66] Halley noted that his method was "a work of too much difficulty for the ordinary Arithmetician to undertake" and he "sought, if it were possible to find a Theorem that might be more concise than the Rules there laid down, but in vain, for all that can be done to expedite it, is by Tables of Logarithms ready computed."[67] Halley's study has been celebrated as a landmark in the history of vital statistics: the field in which the Hollerith electromechanical tabulator (repurposed by LJ Comrie for astronomical work), was invented and first deployed.[68]

The *par excellence* logarithmic institution of the seventeenth century was the Board of Excise. First established during the early years of the long parliament in 1643, the Board of Excise was a national

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[64] William Deringer, "Just Fines: Mathematical Tables, Church Lands, and the algorithmic ethic circa 1628" (unpublished, 2020), 66.

[65] See Jacob Bernoulli, "Some questions about interest, with a solution of a problem about games of chance, proposed," *Journal des Savants* (1685). Jacob Bernoulli also discussed this matter in his 1703-1704 correspondence with Leibniz, alongside commentary on mortality statistics and annuity prices. See Lorraine Daston, *Classical Probability in the Enlightenment* (Princeton: Princeton University Press, 1988), 129.

[66] PGM Dickson, The Financial Revolution in England: a Study in the Development of Public Credit, 1688-1756 (New York: Routledge, 2016), 53.

[67] Edmund Halley, "Some Further Considerations on the Breslaw Bills of Mortality, By the same Hand with the former," *Philosophical Transactions of the Royal Society*, vol. xvii, 1693.

[68] See Patrick Graham, *Biopolitics and the History of the American Electronic Data Processing Industry*, available on request. Harald Westegaard has called Halley's work "seven-league stride" ahead of its precursor—John Graunt's 1662 *Natural and Political Observations upon the Bills of Mortality*. See Harald Westegaard, "On the History and Prospects of Vital Statistics," *Economica*, no. 14 (1925), 123.

computing organization essentially responsible for volumetric measurements of a huge number of barrels. The technique by which this was affected—barrel gauging—was essentially logarithmic. Logarithmic tables for simplifying multiplication and division into addition and subtraction made such volumetric determinations significantly more computationally feasible for the hundreds of rank and file excisemen who were responsible for conducting such measurements, particularly at a time when common techniques in multiplication and division were known to be difficult to learn.[69] Indirect taxation (primarily excise but including customs) has been celebrated by historians as one of the foundational institutions of the British "fiscal-military state," and represented the majority of the English state's revenue during the 50 year span between 1661 and 1714, reaching a height of 80% of total government income during the pivotal years of 1686-88.[70] This transition away from direct taxes (principally land tax) and towards indirect taxation (customs and excise) that began with the Restoration in 1660 would last until the issuance of income taxation during the Napoleonic wars—a substantial development in the long history of the English state, and one that I argue was fundamentally enabled by the emergence of logarithmics.[71]

I thus believe that the general frame of feasible computing, and logarithmics specifically, should be used to link the history of seventeenth-century mathematical physics to the history of the seventeenth-century English state. This would tie these two milestones in the history of modernity together, with a focus on political economy as a hinge between the two. Steven Pincus has forcefully made a case for considering 1688 as the year of the first modern revolution, and has claimed that this period represented an "epochal break in the construction of the state, and perhaps only the state."[72] Building on Pincus's argument, I would want to demonstrate (in future work) how developments in logarithmics tethered this history of the state to the history of science, and connect developments in both of these domains to changing social and cultural attitudes towards practices of quantification. While the revolution of 1688

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[69] Keith Thomas, "Numeracy in Early Modern England: The Prothero Lecture," *Transactions of the Royal Historical Society*, vol. 37 (1987).

[70] John Brewer, *The Sinews of Power: War, Money and the English State, 1688-1783* (London: Unwin Hyman, 1989). For the figures, see Michael J. Braddick, *The Nerves of State: Taxation and the Financing of the English State, 1558-1714* (Manchester: Manchester University Press, 1996), 10.

[71] Patrick K. O'Brien and Phillip A. Hunt, "The Rise of a Fiscal State in England, 1485-1815," *Historical Research*, vol. 66, no. 160 (1993).

[72] Steven Pincus, *1688: The First Modern Revolution* (New Haven: Yale University Press, 2009), 9.

might be conceived of as an apotheosis, the entirety of the seventeenth century—from the death of Elizabeth in 1603 through the founding of the Bank of England in 1694—represents a series of linked developments that together created the modern English state.

The Civil War of the 1640s, and the periods of significant upheaval and instability that directly preceded and followed it, represent an important setting for understanding 1688. Hobbes remarked that "if in time, as in place, there were degrees of high and low, I verily believe that the highest of time would be that which passed between 1640 and 1660…. wherein men used to see best into good and evil."[73] Surely a history of seventeenth-century England should concern itself with these crucial decades, as the activities of the Long Parliament, such as the formation of the excise board in 1643, played key roles in later developments; the loan that founded the Bank of England itself was secured on excise revenue, for example.

But how far back must one go to get a full sense of the developments of the second half of the century? Answering this question requires forming an opinion on one of the most hotly contested territories in English historiography—the chronology and causes of the Civil War. The original post-WWII debates about this topic were principally enacted between the liberal (sometimes called Whig) historians working in the tradition of Macauley, such as Margaret Judson, and the social historians such as Tawney and the Marxist Christopher Hill. Both groups seemed to believe that "a long-drawn-out and increasingly bitter constitutional crisis, lasting on and off from 1606 to 1640" was the basic format of periodization for the period, although they disagreed significantly about who was fighting over what and why.[74] The general agreement between these two camps on a longer, multi-decade timeframe focused on the structural conditions of the civil war can be lumped together as one pole of the temporal debate, despite significant internal disagreement over the specifics. The other pole, a revisionist camp developed through the encouragement of Geoffrey Elton and including historians such as Conrad Russell and Kevin Sharpe, argued that there were no structural causes of the Civil War at all. Ronald Hutton, summarizing

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[73] Thomas Hobbes, *Behemoth: The History of the Causes of the Civil Wars of England, and the Counsels and Artifices by which They Were Carried on from the yer 1640 to the year 1660*, ed. F. Tönnies (London: 1889).

[74] Lawerence Stone, "The Revolution over the Revolution," *The New York Review of Books* (June 11, 1992).

the revisionist position, claimed instead that the Civil War was "a fortuitous accident, unrelated to fundamental political and social processes (as recent historiography has generally indicated it was)."[75] These scholars claimed that an exhausting reading of archival material from the 1620s demonstrated a complete absence of substantial contemporary debate regarding the relationship between King and Parliament, and that such a lack suggested that the causes of the civil war were immanently located in the 1630s, and not previously present.[76] Thus, determining the temporal bounds of my study requires some deliberation between these two positions: were developments of the first decade of the seventeenth century relevant to the spiraling crisis of the 1640s? Or were the historical horizons much more limited?

As Stone notes in his excellent review of these debates, the revisionist position was not tenable for very long. By the late 1980s, a new generation of counter-revisionists had begun to take the short-term view of the Civil War to task. Pointing out that "when the punishment for talking about liberty might be a loss of ears or a life in prison, it is highly misleading to deduce consensus from silence," they challenged the revisionist focus on manuscript material. Taking an interest in popular politics, along with key findings—such as the Parliamentary debates over the Petition of Right in 1628—the counter-revisionists concluded that even in the early decades of the century "there was a large and very interested political nation out there, which the Revisionists had failed to notice."[77] My work in the history of computing has led me to look more favorably on the long, structural view of the causes of the Civil War.

The point of this foray into English political history is to demonstrate the significant temporal overlap that it shares with the history of feasible computing. The logarithmic spirit came to England from Scotland along with the court of King James, and by the end of his reign had found its footing in scientific, administrative and commercial contexts. In the 1620s and 1630s, questions of computational feasibility became central to increasingly rigorous practices in estate accounting, as the royal household lurched away from the use of roman numerals and towards the arabic. The 1640s had the Civil War, and

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[75] Mark E. Kennedy, "Legislation, Foreign Policy, and the 'Proper Business' of the Parliament of 1624," *Albion*, vol. 23, no. 1 (1991), 43.

[76] Lawerence Stone, "The Revolution over the Revolution," *The New York Review of Books* (June 11, 1992).

[77] Lawerence Stone, Ibid.

the Long Parliament's formation of the Board of Excise is, as I have said, a site of supreme interest to the history of feasible computing. The complicated dance between political economy and apocalyptic prophecy was a central feature of Cromwell's foreign policy, and feasible computing played a role in both of these domains (recall the 1645 reprint of Napier's *Plaine Discovery*). The Restoration brought with it Political Arithmetic, and William Petty's reduction of all things to number, weight and measure was only made possible through feasible approximations. By the 1680s—the time of the Glorious Revolution in government and Newton's contemporary invention of universal gravitation—logarithmics had become an essential component of both English state and English science. Unpacking this braided development requires one to be familiar with the general historiography in both of these domains.

A fair number of the classic arguments about the origins of modernity include some role for practices of calculation, and more recent engagements with these scholars have tended to reaffirm the importance of computing to the construction of the modern world. My work can help bring renewed attention to the relationship between computing techniques and the emergence of modernity. I am in good company in my suspicions that some fundamental piece of the riddle of modernity can be uncovered in the history of seventeenth- and eighteenth-century computational practice. Marx noted in the *Communist Manifesto* that the bourgeoise had "drowned the most heavenly ecstasies of religious fervor, of chivalrous enthusiasm, of philistine sentimentalism, in the icy water of egotistical calculation."[78] Adorno and Horkeimer echo this sentiment a century later, emphasizing the nefarious link between egotistical calculation and the spirit of "instrumental reason" which they claimed pervaded Enlightenment thought.[79] Habermas, writing with a distinctly more positive view of legacy of the Enlightenment, reserved a special place for these calculations in his story of the emergence of the modern distinction between public and private, economy and *oikos*.[80] Heidegger, in "The Question Concerning Technology," identified calculative practices as the heart of the modern technological world characterized by what he calls

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[78] Karl Marx and Frederick Engels, *Manifesto of the Communist Party and Its Genesis* (Pacifica: Marxists Internet Archive, 2010), 30.

[79] Max Horkheimer and Theodor W. Adorno, *Dialectic of Enlightenment: Philosophical Fragments* (Stanford: Stanford University Press, 2002).

[80] Jürgen Habermas, *The Structural Transformation of the Public Sphere* (Cambridge: MIT Press, 1962).

"enframement."[81] Weber thought that calculation had a special role to play in the apparatus of bureaucratic power that came to replace the social hierarchies previously dominated by charisma, and Theodore Porter has made similar arguments more recently for the role played by "mechanical objectivity" in granting social authority to calculative formulas.[82] Inspired by Foucault sweeping remarks, I believe that this long history of computing plays a decisive role in story of what he called governmentality and the related concept of biopolitics.[83]

Even more traditionally liberal thinkers, like Joel Mokyr or Douglass North, insist on the essential link between calculation and political economy as it was nurtured by burgeoning institutions of the early modern world.[84] This was not new for that tradition either – the great political economists of the age of Revolutions, Adam Smith and Guillaume Thomas Raynal, had already identified arithmetical practices as key components of the new commercial world.[85] The nineteenth-century utilitarians (Babbage among them) accelerated those liberal interests in computation while cementing the reputation of the new discipline of statistics as that "moral science," equally at home in matters of state or in questions of natural philosophy.[86]

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[81] Martin Heidegger, *The Question Concerning Technology and Other Essays* (New York: Garland Publishing, 1977).

[82] Tony Waters and Dagmar Waters, *Weber's Rationalism and Modern Society: New Translations on Politics, Bureaucracy and Social Stratification* (New York: Palgrave MacMillan, 2015), especially Chapter 6, "Bureaucracy," 73-125. See also Theodore Porter, *Trust in Numbers: The Pursuit of Objectivity in Science and Public Life* (Princeton: Princeton University Press, 1995).

[83] Michel Foucault, *Society Must Defended: Lectures at the College De France, 1975-76* (New York: Picador, 2003), *Security, Territory, Population, Lectures at the College De France, 1977-78* (New York: Palgrave Macmillan, 2009), *The Birth of Biopolitics: Lectures at the College De France, 1978-79* (New York: Palgrave Macmillan, 2008).

[84] Joel Mokyr, *The Enlightened Economy: Britain and the Industrial Revolution, 1700-1850* (New York: Penguin Books, 2009), esp. chapter 4 and chapter 17, and Douglass C. North, *Institutions, Institutional Change and Economic Performance* (Cambridge: Cambridge University Press, 1990).

[85] See, for example Adam Smith's discussion of enumerated and non-enumerated commodities, *An Inquiry into the nature and causes of the Wealth of Nations* (Oxford: Oxford University Press, 1976), Book IV, Chapter VII. Rayanl claimed that "the same understanding that Newton had to calculate the motion of the stars, the merchant exerts in tracing the progress of the commercial people that fertilize the earth."Guillaume-Thomas-François Raynal, *A Philosophical and Political History of the Settlements and Trade of Europeans in the East and West Indies* (London: A. Strahan, 1788), 191.

[86] Ian Hacking, *The Taming of Chance* (Cambridge: Cambridge University Press, 1990).

While many in the history of science have long suspected that calculative practices played a role in building modern science, linking the history of these scientific concerns with a broader history of the relationship between computation and modernity could do much to reinforce the hypothesis that modern science is tied, in fundamental ways, to modern political economy, and thus to both the modern state and the modern market. Such a hypothesis calls for these three arenas of modernity—science, capitalism and the state—to be investigated as a joint phenomenon.[87] Such a project would contribute to an understanding of the important but often underexplored role played by the history of calculation in the development of a number of prominent "meta-narratives" of modernity, as well as offering opportunities to connect my historical research to these larger questions in philosophy and the social sciences.

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[87] Hal Cook has made a powerful argument along these lines for the Dutch case, although he was more focused on "objectivity" and materiality than on mathematics and prediction. I am interested in bringing these two dimensions together in future work, particularly regarding Dutch/English exchange in the 17th century. See Harold Cook, *Matters of Exchange: Commerce, Medicine, and Science in the Dutch Golden Age* (New Haven: Yale University Press, 2008).